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REPORT ON

EXPRESSING MASS UNBALANCE

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EXPRESSING MASS UNBALANCE

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Abstract

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Summary

Three different methods of expressing mass unbalance are presented. These methods are: (1) force vectors in two planes perpendicular to the spin axis, (2) principle axis shift and tilt, (3) force and moment vectors relative to the c.g.

Each method is discussed briefly. However, method (3) above has been chosen as the more useful in expressing mass unbalance for spacecraft payloads, because:

a. Specify payload data, other than c.g. location, is not required.

b. Complete separation of static and dynamic unbalance is provided.

c. Convenient comparison to other payload configurations is allowed.

Since unbalance information is conventionally recorded in two planes perpendicular to the spin axis, a unique conversion operation is presented which allows (1) to be expressed as (3).

EXPRESSING MASS UNBALANCE

Purpose

The purpose of this report is to clarify the specification of static and dynamic mass unbalance, as related to balancing of spacecraft payloads.

Introduction

The conventional dynamic balancing machines have force or velocity type pickup devices in two planes perpendicular to the spin axis. The recordings obtained in these planes will be called force vector indications. It is considered here that a force vector consists of a weight times its distance from the spin axis, and a moment vector consists of a weight times its distance from the spin axis times a distance along the spin axis.

Methods Of Expressing Mass Unbalance

The most fundamental method of expressing dynamic unbalance is that which results in a force vector in each of two planes perpendicular to the spin axis. No specific payload data is needed, since this method of unbalance expression is not a function of payload weight, c.g. location, or mass moment of inertia. However, this method does not allow different payload configurations to be compared conveniently, and it does not readily resolve static from dynamic unbalance.

Static and dynamic unbalance may also be expressed as principle axis shift and tilt, respectively.[1] The static unbalance is computed with the following equation:

$$S = W\rho \qquad (1)$$

where: S = static unbalance
W = payload weight
ρ = principle axis shift

This can be visualized best by studying the motion of a spinning body just separated from the last stage of a rocket vehicle. Before separation, the payload is spun-up about the spin axis of the vehicle spin table, which does not necessarily correspond with the principle axis of the payload. After separation, the payload will spin about its principle axis. The distance between the payload spin axis before separation and after separation is called the principle axis shift. In other words, when rotating in space, the payload spin axis existing before separation will generate a cylinder about the payload spin axis after separation. The radius of this generated cylinder is the principle axis shift.

1. Shift and tilt with respect to the spin axis

The dynamic unbalance is computed with the following equation:[2]

$$D = \alpha g \, (I_x - I_z) \text{ - - - - - - - - - - } (2)$$

where:
D = dynamic unbalance
α = semiconing angle
g = gravatational constant
I_x = mass moment of inertia of about yaw axis
I_z = mass moment of inertia about spin axis

Studying the motion of a payload immediately after separation from a rocket vehicle, and considering only the dynamic unbalance forces (i.e., neglect kick-off forces, gyroscopic forces, and any other nutation producing forces), the payload principle axis will generate a double cone with the common apex of the double cone located at the c.g. of the payload. The angle of the generated cone will be 2α.

In a realistic case of unbalance, there will exist a combination of static and dynamic unbalance (i.e., both a principle axis shift and tilt, superimposed).

This method of expressing dynamic unbalance requires knowing the moment of inertia of both spin and yaw axes, and the payload weight. However, it does permit a comparison to other payload configurations, and it clearly separates static from dynamic unbalance.

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2. Assuming $\tan\alpha = \alpha$, D is small compared to I_x and I_z, and $I_x \neq I_z$

The third method of expressing static and dynamic unbalance relates force and moment vectors to the c.g. of the payload. The only specific payload data needed is the c.g. location along the spin axis. Since this method permits separation of static and dynamic unbalance, and allows a convenient comparison to other payload configurations, an operation has been devised which converts force readings in two planes perpendicular to the spin axis into force and moment vectors relative to the c.g. of the payload. After these values have been obtained, direct substitution may be made into equation (1) and (2) to determine principle axis shift and tilt.

Method For Conversion Of Force Vectors In Two Planes Perpendicular To The Spin Axis Into Force And Moment Vectors Relative To The Payload C. G.

The unbalance forces indicated in any two predetermined planes, perpendicular to the spin axis, will be defined as the near plane unbalance force vector, N, and the far plane unbalance force vector, F. For the case illustrated in Figure 1, it is assumed that the plane of the center of gravity is located between the near and far planes. The end view in Figure 2 illustrates the technique by which the near and far plane force vectors are resolved into their respective static and dynamic components. Consider the following:

1. Perform vectorial addition of F and N to yield S.
2. Extend S as necessary
3. Construct a line perpendicular to S (or S extended) and through F and N.

Now F and N may be resolved into their static and dynamic force vectors. The static force vectors, N_s and F_s are colinear with the static resultant, S, and the dynamic force vectors, N_d and F_d, are in opposite directions and are perpendicular to the static resultant, S.

It may be seen in Figure 1 that the vectorial addition of N_s and F_d yield S, the static resultant. Its position along the spin axis will be determined by the summation of moments for static equilibrium. This method, by taking moments about the near plane, yields:

$$l_s = \left(\frac{F_s}{S}\right) l \qquad (3)$$

where: l = distance from near plane to far plane
l_s = distance from near plane to static resultant

Since the magnitudes of S, F_s and l are known, l_s may be readily computed.

The dynamic force vector F_d and N_d, being equal in magnitude, and opposite in direction and separated from each other by the distance l, constitute a pure dynamic couple. This couple may be resolved into a vector by the right hand rule, and will be called a pure dynamic unbalance moment vector, D_p. Its value is N_d (or its equal F_d) X l. Since a moment vector can be moved from one station to any other parallel station without changing its effect, D_p will be placed at the center of gravity of the system (as shown in the c.g. plane in Figure 1).

Now, since the static resultant, S, is not located at the c.g., it produces a moment about the c.g. This effect may be referred to as an induced dynamic unbalance. To arrive at its magnitude and direction, place both the +S and -S in the plane of the static resultant at the c.g. This operation reveals an induced dynamic unbalance moment vector, D_i, caused by the static resultant, S, being off the c.g; and permits translation of the static resultant, S, to the c.g. The magnitude of D_i equals $S(\left| l_s - l_{cg} \right|)$, and applying the right hand rule, this moment vector will be perpendicular to the plane of the static resultant, and at right angles to the pure dynamic moment vector, D_p.

To Review: All forces have been transferred to the c.g. plane.

a. Static resultant force vector, S, which is in the plane of the static resultant.

b. Pure dynamic moment vector, D_p, which is in the plane of the static resultant.

c. Induced dynamic moment vector, D_i which is perpendicular to the plane of the static resultant.

d. The value of the total dynamic moment vector, D, equals:

$$D = \sqrt{D_p^2 + D_1^2} \quad - - - - - - - - (4)$$

The angular, of phase portion of S and D may be readily determined by the principles of statics. Figure 3 is an actual graph, showing the application of the above described method.

This method is usually applied so as to resolve the near and far plane vectors into their equivalent force and moment vectors at the c.g., but is not restricted to this location. The same method may be used to determine the equivalent force and moment vectors at any arbitrary position along the spin axis. It could be used to determine the unbalance at the spacecraft interface, and then it would not even be necessary to know the location of the c.g. in the payload.

The operation has been found simple to apply and is useful in converting the data - both initial and residual unbalance vectors, which are available from balance machine operations - to a form which is readily interpreted, and convenient for further operation.

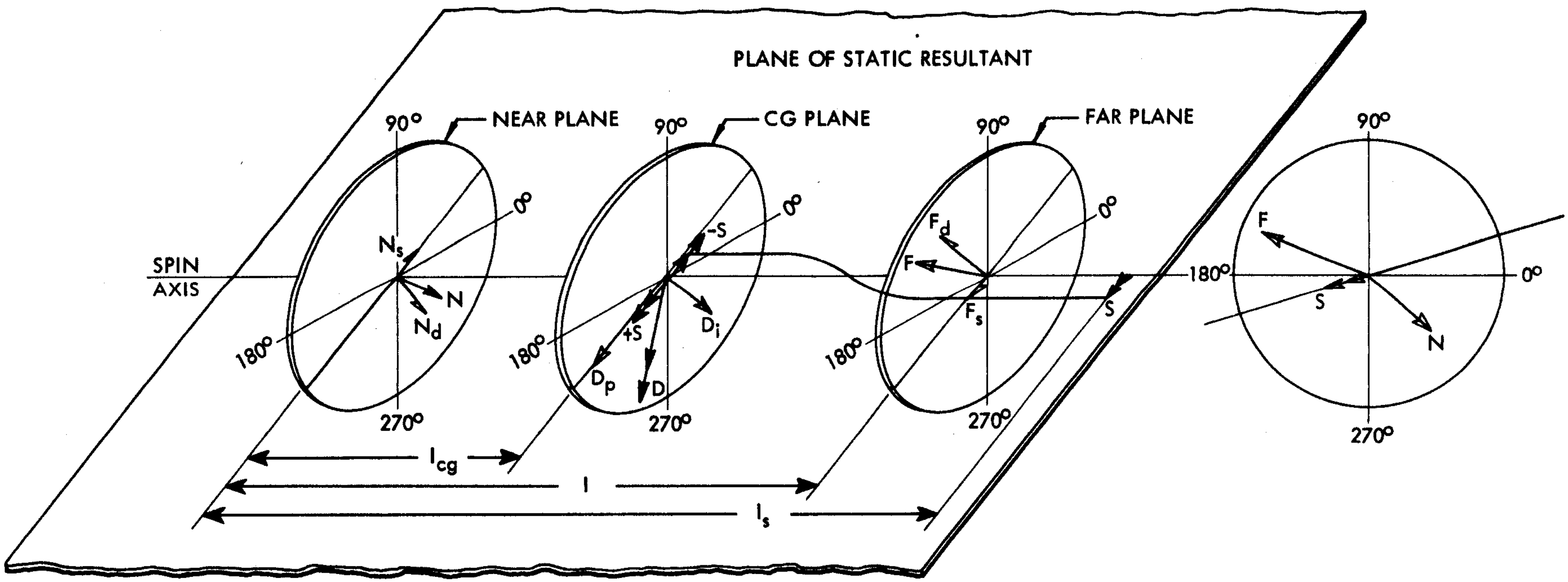

Figure 1. Transfer of Unbalance Force Vectors in Near and Far Planes into Unbalance Force and Moment Vectors in cg Plane.

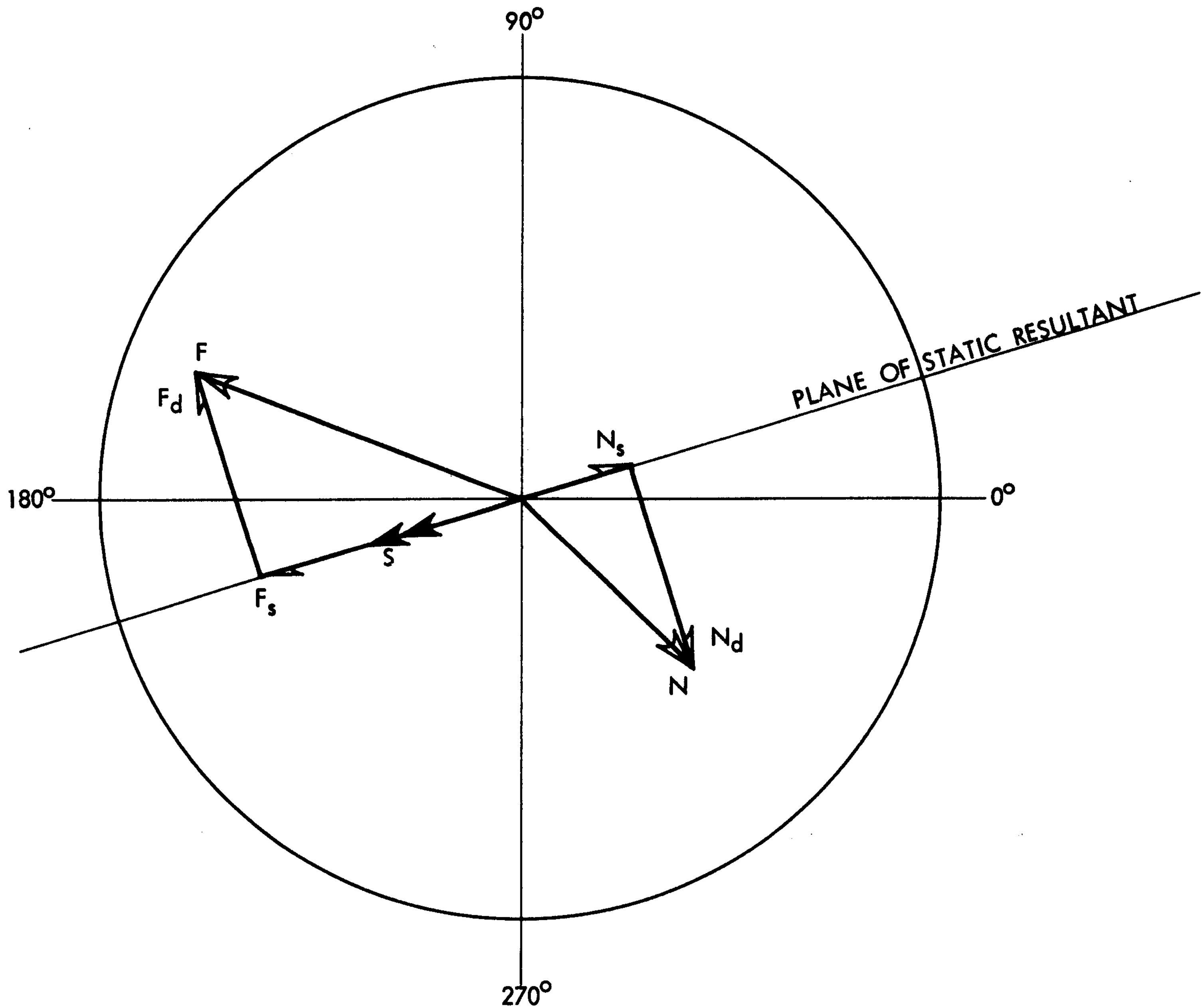

Figure 2. Resolution of Near and Far Plane Unbalance Force Vectors Into Static and Dynamic Components.

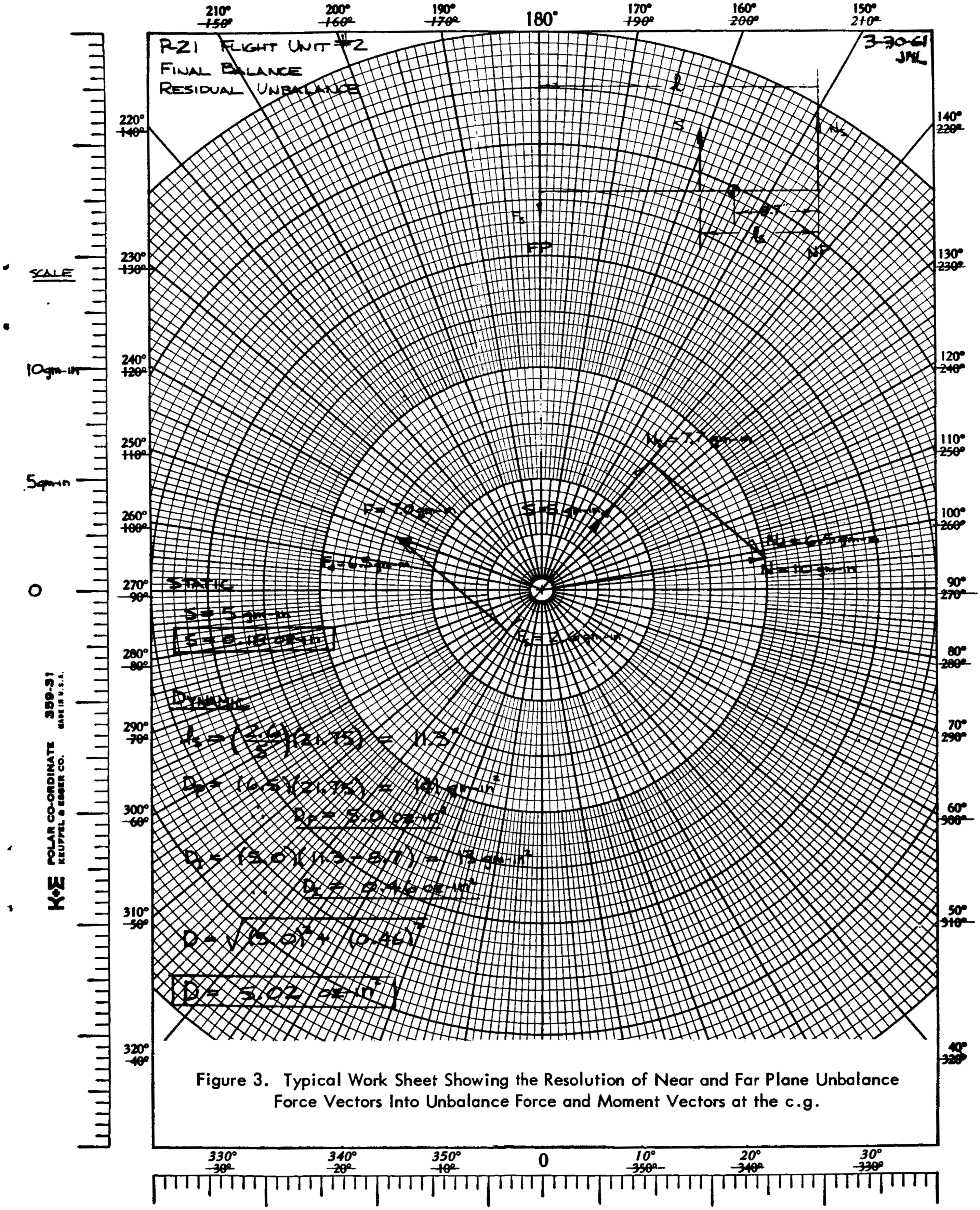

Figure 3. Typical Work Sheet Showing the Resolution of Near and Far Plane Unbalance Force Vectors Into Unbalance Force and Moment Vectors at the c.g.